Why Students Struggle with Mathematics
Gloria Maccow, Ph.D., and Adam Scheller, Ph.D., NCSP

Why Students Struggle with Math: A Look at Math-Related Processes
Gloria Maccow, Ph.D.
Adam Scheller, Ph.D., NCSP

Previous Webinars in the Series

- Why Students Struggle to Learn: An Overview of Cognitive Factors
  Original Air Date: December 14, 2011
  Presenter: Gloria Maccow, PhD

- The Role of Attention and Executive Functioning in the Process of Learning
  Original Air Date: January 18, 2012
  Presenter: Adam Scheller, PhD

- Why Some Students Struggle with Reading: A Look at Reading-Related Processes
  Original Air Date: March 29, 2012
  Presenters: Karen Apgar, MA CAGS/NCSP,
  Gloria Maccow, PhD and Justin Potts, MS NCSP

Objectives

- Describe the cognitive processes and abilities involved in the acquisition of mathematics skills.
- Illustrate how we use assessment data to
  - document under-achievement in mathematics,
  - determine which mathematics-related processes are linked to the under-achievement, and
  - identify appropriate interventions.

Learning Mathematics . . .

. . . requires continual integration and coordination of conceptual and procedural knowledge.

(Rittle-Johnson, Siegler, Alibali, 2001)

349 + 23 = _____

+ 3 4 9
5 2 3
Math-Related Cognitive Processes

Working Memory Theory

(Baddeley, 2003)
Coping Alphanumeric Stimuli in Working Memory (Beminger, 2007)

Alphanumeric Stimuli Stored in Orthographic or Numeric Working Memory

Phonological Loop

Orthographic Loop

Executive Functions—Switching Set (Beminger, 2007)

RAS
Switching Stimuli Samples

Word-Digit-Word
Digit-Word-Digit
Word-Digit-Word

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### Cognitive Processes

- Coding in working memory of visual symbols of numbers (numerals)
- Writing numeric symbols
- Making automatic associations of verbal names with visual numerals
- Accessing math facts in long-term memory (LTM)
- Visual-spatial and temporal-sequential aspects of computational operations
- Executive monitoring of math operations
- Reasoning with numbers to solve math problems
- Holding in memory quantitative, visual-spatial, and verbal information while working on the problem

### Mathematics Skills and Mathematics-Related Competencies and Processes
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Math Skills and Supporting Competencies

- Oral Counting
- Math Computation (Basic Arithmetic)
- Math Fact Retrieval
- Computational Operations

Conceptual Knowledge
- Place Value
- Part-Whole Relationships

Math Problem Solving
- Representing problems
- Planning steps for problem solution
- Self-monitoring

Math Skills and Supporting Competencies

Part-Whole Relationships

- 2
- 1/2

- 4
- 1/4

<table>
<thead>
<tr>
<th>Mathematical Domain</th>
<th>Supporting Competencies</th>
<th>Underlying Cognitive Systems</th>
<th>Language System</th>
<th>Visuospatial System</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e.g., Base-10 Arithmetic)</td>
<td>Conceptual (e.g., base-10 knowledge)</td>
<td>Central Executive</td>
<td>Information Representation</td>
<td>Information Representation</td>
</tr>
<tr>
<td></td>
<td>Procedural (e.g., columnar adding)</td>
<td>Attentional and Inhibitory Control of Information Processing</td>
<td>Information Manipulation</td>
<td>Information Manipulation</td>
</tr>
</tbody>
</table>

#### Case Example

**Introducing Steven**

(Adapted from WISC-IV Integrated Working Memory Domain Case Study)
Background Information

- Meet Steven . . . a 12-year-old boy in the 6th grade. He enjoys science and eagerly talks about research that he is conducting on various topics. He appears to enjoy one-on-one time with adults when he is allowed to discuss topics of interest to him.
- Steven is inquisitive and likes to figure out how things work. He is fascinated by clocks and likes to take them apart.

Background Information

- Steven is often impulsive, especially when he sees something that appeals to him. He has trouble following directions, and frequently requires them to be repeated.
- He has been experiencing behavior problems at school. The teacher describes Steven as forgetful and says that he needs frequent reminders to stay on task in the classroom.
Background Information

In school, Steven is struggling to comprehend grade-level text. His performance on writing and math assignments is below that of his peers. His teachers have observed that Steven is often talking when he should be working.

Background Information

• His parents have observed similar behaviors at home. They note that Steven often does not remember his chores. When he does remember, he tends to stop before the tasks are completed. He complains about homework and refuses at times to complete written assignments.

• Both parents and teachers describe Steven as a bright young boy who needs to work harder on school work.
Background Information

- Steven is being assessed to determine if his difficulties with following directions are because of his current cognitive functioning.
- Another goal of the assessment is to determine if any of Steven’s academic challenges are severe enough to require individualized, intensive intervention.
- The discussion of the assessment results will focus on math.

Why is Steven Struggling to Master Grade-Level Objectives in Math?
### Ability—WISC-IV

(S and W are Personal Strengths and Weaknesses)

<table>
<thead>
<tr>
<th>Index/Subtest</th>
<th>Composite/Scaled Score</th>
<th>Index/Subtest</th>
<th>Composite/Scaled Score</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Verbal Comprehension</strong></td>
<td>114</td>
<td><strong>Perceptual Reasoning</strong></td>
<td>125</td>
</tr>
<tr>
<td>Comprehension</td>
<td>15</td>
<td>Block Design (W)**</td>
<td>11</td>
</tr>
<tr>
<td>Similarities</td>
<td>13</td>
<td>Matrix Reasoning (S)**</td>
<td>17</td>
</tr>
<tr>
<td>Vocabulary (W)**</td>
<td>10</td>
<td>Picture Concepts</td>
<td></td>
</tr>
<tr>
<td><strong>Working Memory</strong></td>
<td>80</td>
<td><strong>Processing Speed</strong></td>
<td>121</td>
</tr>
<tr>
<td>Digit Span (W)*</td>
<td>6</td>
<td>Coding</td>
<td>12</td>
</tr>
<tr>
<td>Letter-Number Sequencing (W)*</td>
<td>7</td>
<td>Symbol Search</td>
<td>15</td>
</tr>
<tr>
<td>(Arithmetic)</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Full Scale IQ = 115; General Ability Index = 123

**Index-level comparisons; *Compared to Overall Mean.**

### Cognitive Strengths and Weaknesses

<table>
<thead>
<tr>
<th>Index Comparisons</th>
<th>Index Score 1</th>
<th>Index Score 2</th>
<th>Diff.</th>
<th>Critical Value (0.05)</th>
<th>Sig. Diff.?</th>
<th>Base Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCI - PRI</td>
<td>114</td>
<td>125</td>
<td>-11</td>
<td>10.18</td>
<td>Y</td>
<td>22.3%</td>
</tr>
<tr>
<td>VCI - WMI</td>
<td>114</td>
<td>80</td>
<td>34</td>
<td>10.18</td>
<td>Y</td>
<td>1.1%</td>
</tr>
<tr>
<td>VCI - PSI</td>
<td>114</td>
<td>121</td>
<td>-7</td>
<td>11.75</td>
<td>N</td>
<td>36.2%</td>
</tr>
<tr>
<td>PRI - WMI</td>
<td>125</td>
<td>80</td>
<td>45</td>
<td>11.00</td>
<td>Y</td>
<td>0.4%</td>
</tr>
<tr>
<td>PRI - PSI</td>
<td>125</td>
<td>121</td>
<td>4</td>
<td>12.47</td>
<td>N</td>
<td>40.9%</td>
</tr>
<tr>
<td>WMI - PSI</td>
<td>80</td>
<td>121</td>
<td>-41</td>
<td>12.47</td>
<td>Y</td>
<td>0.8%</td>
</tr>
<tr>
<td>FSIQ - GAI</td>
<td>115</td>
<td>123</td>
<td>-8</td>
<td>3.73</td>
<td>Y</td>
<td>16.1%</td>
</tr>
</tbody>
</table>
More on Working Memory
(WISC-IV Integrated Scores)

<table>
<thead>
<tr>
<th>Working Memory Domain Subtest</th>
<th>Scaled Score</th>
<th>Percentile Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digit Span Forward</td>
<td>7</td>
<td>16</td>
</tr>
<tr>
<td>Digit Span Backward</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Spatial Span Forward</td>
<td>11</td>
<td>63</td>
</tr>
<tr>
<td>Spatial Span Backward</td>
<td>12</td>
<td>75</td>
</tr>
</tbody>
</table>

WAT-III Composite Score Summary

<table>
<thead>
<tr>
<th>Composite</th>
<th>Standard Score</th>
<th>95% Conf. Interval</th>
<th>%ile Rank</th>
<th>Qualitative Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Reading</td>
<td>82</td>
<td>77–87</td>
<td>12</td>
<td>Below Average</td>
</tr>
<tr>
<td>Basic Reading</td>
<td>82</td>
<td>78–86</td>
<td>12</td>
<td>Below Average</td>
</tr>
<tr>
<td>Reading Comp. and Fluency</td>
<td>86</td>
<td>78–94</td>
<td>18</td>
<td>Average</td>
</tr>
<tr>
<td>Written Expression</td>
<td>81</td>
<td>74–88</td>
<td>10</td>
<td>Below Average</td>
</tr>
<tr>
<td>Mathematics</td>
<td>84</td>
<td>78–90</td>
<td>14</td>
<td>Below Average</td>
</tr>
<tr>
<td>Math Fluency</td>
<td>85</td>
<td>78–92</td>
<td>16</td>
<td>Average</td>
</tr>
</tbody>
</table>
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Selected Subtest Scores

<table>
<thead>
<tr>
<th>Subtest</th>
<th>Standard Score</th>
<th>95% Conf. Interval</th>
<th>%ile Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Problem Solving</td>
<td>92</td>
<td>84–100</td>
<td>30</td>
</tr>
<tr>
<td>Numerical Operations</td>
<td>79</td>
<td>73–85</td>
<td>8</td>
</tr>
<tr>
<td>Math Fluency Addition</td>
<td>87</td>
<td>76–98</td>
<td>19</td>
</tr>
<tr>
<td>Math Fluency Subtraction</td>
<td>86</td>
<td>76–96</td>
<td>18</td>
</tr>
<tr>
<td>Math Fluency Multiplication</td>
<td>86</td>
<td>76–96</td>
<td>18</td>
</tr>
</tbody>
</table>

Eligibility and Need For Direct Specialized Instruction
Determining the Existence of a Specific Learning Disability
### Specific Learning Disability

#### IDEA 2004

...a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations, ...  

§ 300.8

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### Specific Learning Disability

#### IDEA 2004

The child does not achieve adequately for the child’s age or to meet State-approved grade-level standards in one or more of the following areas, when provided with learning experiences and instruction appropriate for the child’s age or State-approved grade-level standards:

<table>
<thead>
<tr>
<th>Oral Expression</th>
<th>Listening Comprehension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Written Expression</td>
<td>Reading Comprehension</td>
</tr>
<tr>
<td>Basic Reading Skills</td>
<td>Reading Fluency Skills</td>
</tr>
<tr>
<td>Mathematics Calculation</td>
<td>Mathematics Problem-Solving</td>
</tr>
</tbody>
</table>

§ 300.309 (a) (1)
Specific Learning Disability
IDEA 2004

The child does not make sufficient progress to meet age or State approved grade-level standards in one or more of the areas identified in paragraph (a)(1) of this section when using a process based on the child’s response to scientific, research-based intervention; or

§ 300.309 (a) (2) (i)

Specific Learning Disability
IDEA 2004

the child exhibits a pattern of strengths and weaknesses in performance, achievement, or both, relative to age, State-approved grade level standards or intellectual development . . . ” (§ 300.309 (a) (2) (ii)).
Specific Learning Disability
IDEA 2004

Exclusionary Factors
- vision, hearing, motor problems
- intellectual disability
- emotional disturbance
- cultural and/or environmental issues
- limited English proficiency
- lack of instruction

Pattern of Strengths and Weaknesses
Pattern of Strengths and Weaknesses

- **Processing Strength**
  - WISC–IV Verbal Comprehension Index
  - SS = 114

- **Processing Weakness**
  - WISC–IV Working Memory Index
  - SS = 80

- **Achievement Weakness**
  - WIAT–III Numerical Operations
  - SS = 79

**Discrepant?**
- A: Yes
- B: Yes

**Pattern of Strengths and Weaknesses Analysis**

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Relative Strength Score</th>
<th>Relative Weakness Score</th>
<th>Diff.</th>
<th>Critical Value .05</th>
<th>Sign. Diff. Y / N</th>
<th>Supports SLD hypothesis?</th>
<th>Yes / No</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Processing Strength / Achievement Weakness</td>
<td>114 (VCI)</td>
<td>79 (NO)</td>
<td>35</td>
<td>8.82</td>
<td>Y</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>B Processing Strength / Processing Weakness</td>
<td>114 (VCI)</td>
<td>80 (WMI)</td>
<td>34</td>
<td>10.18</td>
<td>Y</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>

The PSW model is intended to help practitioners generate hypotheses regarding clinical diagnoses. This analysis should always be used within a comprehensive evaluation that incorporates multiple sources of information.
Why Did Steven Struggle with Basic Math Operations?

See Numerical Operations WIAT-III

Written Calculation Requires:

- Counting fluency.
- Quantitative working memory.
- Numeric coding of number symbols in working memory.
- Phonological and orthographic loops in working memory.
- Numeral writing.
- Oral and written fluency in retrieving math facts from memory.
- Spatial alignment of numbers on the page.
- Application of algorithms—sequential computation steps.
- Place value concept.
- Part-whole concept.
- Executive functions for mental set switching set and self-monitoring.
Summary of Findings

- Steven’s achievement in reading, mathematics, and writing is a weakness relative to his higher-order reasoning abilities.
- His achievement is adversely affected by weaknesses in registration and manipulation of auditory information.
- His achievement in basic mathematics skills is adversely affected by weaknesses in retrieving math facts from long-term memory.

Recommendations

- Steven’s performance in academic areas was affected adversely by weaknesses in registration and manipulation of auditory information. Therefore, we recommend assessment of his auditory processing abilities.
- Further assessment of attention and executive functioning will provide information on his strengths and needs in these domains. Information on cognitive efficiency will be helpful for instructional planning.
**Recommendations**

- Interventions for math should focus on improving accuracy and fluency of basic facts.

- A calculator may be helpful when Steven is completing mathematics reasoning tasks.
Improving Automaticity

To help Steven to commit math facts to automatic recall, allow him to practice them with different combinations of input (looking or listening) and output (writing or saying).

- Present the math fact visually. Steven looks at the math fact and then writes the sum, difference, product, or quotient.
- Present the math fact visually. Steven looks at the math fact and then says the sum, difference, product, or quotient.
- Present the math fact audibly. Steven listens to the math fact and then writes the sum, difference, product, or quotient in writing.
- Present the math fact audibly. Steven listens to the math fact and then says the sum, difference, product, or quotient in writing. (Berninger, 2007).

Short-term Memory Interventions

Most interventions to improve short-term memory involve rehearsal training.

Rehearsal Strategies

- Say the material over and over to oneself.
- Engage in serial repetition. This allows information to be maintained in WM for longer periods of time, thus enhancing short-term recall. Elaborative rehearsal also facilitates long-term storage.
Verbal Working Memory Interventions

Chunking
- Pairing, clustering, grouping, or association of different items into units that are processed and remembered as a whole. This facilitates short-term retention and consolidation into long-term storage.

Paraphrasing
- A strategy that builds on both rehearsal and chunking. Students restate information in their own words. This requires that they reorganize and condense a large amount of linguistic information into smaller, well-integrated, and more personally meaningful units.

Executive Working Memory Interventions

Dual Encoding
- Strategies utilizing concurrent visual and verbal encoding.
- Some dual encoding occurs naturally (e.g., number naming).
- In the classroom, visual and verbal materials should be utilized.
**Executive Working Memory Interventions**

**Organizational Strategies**

- Fitting existing information into an organized structure (semantic category).
- Structuring and organizing information reduces the processing load on WM, thereby allowing more efficient encoding.
- Organizing information involves rehearsal and the processing of information at a deeper level.

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**Working Memory Training**

An Evidence-based intervention for working memory training.

www.cogmed.com
References


References


References
