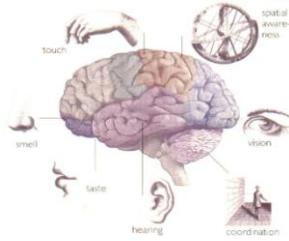


# THE NEUROPSYCHOLOGY OF MATHEMATICS



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## Presentation Goals

- (1) Discuss the international trends in math, and reasons why the United States lags behind many industrialized nations in math and science.
- (2) Introduce a *brain-based* educational model of math by identifying three basic neural codes which format numbers in the brain.
- (3) Explore the role of various cognitive constructs including working memory, visual-spatial functioning, and executive functioning, with respect to math problem solving ability.
- (4) Introduce the 90 minute assessment model of mathematics and interventions.

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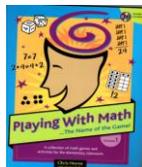
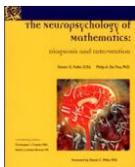
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## Future Reading Materials



www.schoolneuropsychpress.com  
or  
@schoolneuropsychpress

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## 2011 TIMSS DATA

[http://nces.ed.gov/pubs2013/2013009\\_1.pdf](http://nces.ed.gov/pubs2013/2013009_1.pdf)

### Trends in International Mathematics and Science Study: (5<sup>th</sup> Sample Collected Since 1995)

**Grade 4:** TIMSS assesses student knowledge in three content domains: number, geometric shapes and measures, and data display. **50%** of items consisted of number content

**Grade 8:** TIMSS assesses student knowledge in four content domains: number, algebra, geometry, and data and chance. **29%** of items consisted of number content.

- ▶ U.S. national sample consisted of 369 schools and 12,569 students at grade 4, and 501 schools and 10,477 students at grade 8. Data was collected from both public and private schools.
- ▶ In 2011, TIMSS was administered at grade 4 in 57 countries and at grade 8, in 56 countries.

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## 2011 TIMSS DATA: Grade 4

- ▶ The U.S. average mathematics score at grade 4 (541) was higher than the international TIMSS scale average, which is set at 500.3.
- ▶ At grade 4, the United States was among the **top 15** education systems in mathematics (8 education systems had higher averages and 6 were not measurably different) and scored higher, on average, than 42 education systems.
- ▶ Compared with 1995, the U.S. average mathematics score at grade 4 was **23** score points higher in 2011 (541 vs. 518).
- ▶ At grade 4, the United States was among the **top 10** education systems in science, and scored higher, on average, than 47 education systems.

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## 2011 TIMSS DATA 4<sup>th</sup> Grade

Country	Average Score
<b>International Average</b>	<b>500</b>
1. Singapore	606
2. Korea	605
3. Hong Kong	602
4. Chinese Taipei	591
5. Japan	585
6. Northern Ireland	562
7. Belgium	549
8. Finland	545
9. England	542
10. Russian Federation	542
<b>11. UNITED STATES</b>	<b>541</b>
12. Netherlands	540
13. Denmark	537
14. Lithuania	534
15. Portugal	532
16. Germany	528
17. Ireland	527
18. Serbia	516
19. Australia	516
20. Hungary	515
21. Slovenia	513
22. Czech Republic	511
23. Austria	508

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## 2011 TIMSS DATA: Grade 8

- ▶ The U.S. average mathematics score at grade 8 (**509**) was higher than the international TIMSS scale average, which is set at **500**.
- ▶ At grade 8, the United States was among the **top 24** education systems in mathematics (12 were not measurably different), and scored higher, on average, than 32 education systems.
- ▶ Among the U.S. states that participated in TIMSS at grade 8, Massachusetts, Minnesota, North Carolina, and Indiana scored above the TIMSS scale average, while Alabama scored both below the TIMSS scale average.
- ▶ Compared with 1995, the U.S. average mathematics score at grade 8 was 17 score points higher in 2011 (509 vs. 492).
- ▶ At grade 8, the United States was among the **top 23** education systems in science, and scored higher, on average, than 33 education systems.

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## 2011 TIMSS DATA 8<sup>th</sup> Grade

Country	Average Score
<b>International Average</b> <span style="float: right;"><b>500</b></span>	
1. Korea	613
2. Singapore	611
3. Chinese Taipei	609
4. Hong Kong	586
5. Japan	570
6. Russian Federation	539
7. Israel	516
8. Finland	514
9. <b>UNITED STATES</b>	<b>509</b>
10. England	507
11. Hungary	505
12. Australia	505
13. Slovenia	505
14. Lithuania	502
15. Italy	498
16. New Zealand	488
17. Kazakhstan	487
18. Sweden	484
19. Ukraine	479
20. Norway	475
21. Armenia	467
22. Romania	458
23. United Arab Emirates	456
24. Turkey	452
25. Lebanon	449

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## PISA DATA (2009): 15 yr. olds (Program for International Student Assessment)

A test of mathematical literacy for **15**-year old students which focuses upon the direct application of mathematical principles. The test is administered every three years, with **65** countries participating in 2009. The test was **not** designed to measure curricular outcomes, but rather to assess mathematics literacy within a real world context.

- ▶ In 2009, the average U.S. score in mathematics literacy was **487**, lower than the international average score of **496**.
- ▶ Among the 65 countries in the sample, the U.S. was outperformed by 23 countries, and 12 countries had average scores not measurably different.
- ▶ There was no measureable change in the U.S. position when compared to the international average in both 2003 to 2006.
- ▶ U.S. boys scored 20 points higher than girls in math literacy (497 to 477)

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### PISA DATA (2009): 15 yr. olds

Country	Average Score
<b>International Average</b>	<b>498</b>
1. Shanghai-China	600
2. Singapore	562
3. Hong Kong-China	555
4. Korea	546
5. Chinese Taipei	543
6. Finland	541
7. Liechtenstein	536
8. Canada	527
9. Netherlands	526
10. Macao-China	525
11. New Zealand	519
12. Belgium	515
13. Australia	514
14. Germany	513
15. Estonia	512
16. Iceland	507
17. Denmark	503
18. Slovenia	501
19. Norway	498

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### PISA DATA (2009): 15 yr. olds

Country	Average Score
<b>International Average</b>	<b>498</b>
20. France	497
21. Slovak Republic	497
22. Austria	496
23. Poland	495
24. Sweden	494
25. Czech Republic	493
26. United Kingdom	492
27. Hungary	490
28. Luxembourg	489
29. Ireland	487
30. Portugal	487
<b>31. UNITED STATES</b>	<b>487</b>
32. Spain	483
33. Italy	483
34. Latvia	482

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- ### 4 Reasons for U.S Decline
1. **The language of math matters!** Building number connections centered around a base-10 principle is crucial in the development of mathematical efficiency when problem solving.
  2. **Dry and boring material.** Mathematical skill building needs to be **FUN**, and therefore needs to be presented in the format of games and activities.
  3. **Too much focus on the answers.** In order to become facilitators of mathematical knowledge, students should practice multiple methods of problem solving from both a visual-spatial and verbal approach.
  4. **Time on task.** Most elementary math instruction occurs in the afternoon, just 45 minutes per day.

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## What is a Math Disability?

### Basic Terminology:

- **Math Disability (Dyscalculia)**- refers to children with markedly poor skills at deploying basic computational and cognitive processes used to solve equations (Haskell, 2000). These may include deficits with:

- (1) Poor language and verbal retrieval skills
- (2) Working memory skills
- (3) Executive functioning skills
- (4) Faulty visual-spatial skills

\* There is no consensus definition of a true math learning disability at this time (Lewis, 2010).

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## The "MLD" Profile

(Geary, 2011; Rasanen, et al., 2009)

1. Are slower in basic numeric processing tasks:
  - Rapidly identifying numbers.
  - Making comparisons between magnitude of numbers.
  - Counting forwards and backwards
2. Struggle in determining quantitative meaning of numbers:
  - Poor use of strategies.
  - Do not visualize numbers well.
3. Have difficulty learning basic calculation procedures needed to problem solve.

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## The "MLD" Profile

(Geary, 2011; Rasanen, et al., 2009)

### MLD Error Profile:

- Prone to procedural errors such as saying "5,6,7" when solving  $5 + 3 = \underline{\quad}$
- Misalign numbers:
 
$$\begin{array}{r} 36 \\ +3 \\ \hline 66 \end{array}$$
- Fail to borrow in a sequential manner:
 
$$\begin{array}{r} 83 \\ -44 \\ \hline 41 \end{array}$$
- Often deploy the wrong computational process:  
 "The school store sold twice as many pencils to Sam than Robert. If Sam was sold four pencils, how many pencils were sold to Robert?" 8
- Poor retrieval of basic facts:  $7 \times 6 = 35$

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## Language and Mathematics

### Language Skills: (temporal lobes)

- ▶ Most Asian languages have linguistic counting systems past *ten* (*ten-one, ten-two, etc*) whereas English deviates from base-10 system (Campbell & Xue, 2001).
- ▶ In English counting system, decades come first then unit (i.e. *twenty-one*) or sometimes this is reversed (i.e. *fifteen, sixteen, etc...*)
- ▶ Chinese numbers are brief (i.e. *4=si, 7=qi*) allowing for more efficient memory. By age four, Chinese students can count to 40, U.S. students to 15.
- ▶ U.S. kids spend **180** days in school  
South Korea children spend **220** days in school  
Japan kids spends **243** days in school

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## Working Memory and Math

### Working Memory System

- *Phonological Loop*
- *Visual-Spatial Sketchpad*
- *Central Executive System*

### Mathematical Skill

- *Retrieval of math facts*
- *Writing dictated numbers*
- *Mental math*
- *Magnitude comparisons*
- *Geometric Proofs*
- *Inhibiting distracting thoughts*
- *Modulating anxiety*
- *Regulating emotional distress.*

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## Interventions for Lower Working Memory

- ▶ Number-line situated on student's desk.
- ▶ Use a calculator.
- ▶ Reduce anxiety in the classroom.
- ▶ Increase number sense through games such as dice, domino's, cards, etc..
- ▶ Encourage paper and pencil use while calculating equations.
- ▶ Use mnemonic techniques to teach math algorithm's and sequential steps to problem solving (i.e. The steps for long division are  
**D**ivide, **M**ultiply, **S**ubtract, **B**ring **D**own:  
**D**ad **M**om **S**ister **B**rother  
**D**ead **M**onkies **S**mell **B**ad

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## Executive Functioning and Mathematics

<u>EXECUTIVE DYSFUNCTION</u>	<u>BRAIN REGION</u>	<u>MATH SKILL</u>
<ul style="list-style-type: none"> <li>• <i>Selective Attention</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Anterior Cingulate/ Subcortical structures</i></li> </ul>	<ul style="list-style-type: none"> <li>• Procedure/algorithm knowledge impaired</li> <li>• Poor attention to math operational signs</li> <li>• Place value mis-aligned</li> </ul>
<ul style="list-style-type: none"> <li>• <i>Visual Planning Skills</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Dorsal-lateral PFC</i></li> </ul>	<ul style="list-style-type: none"> <li>• Poor estimation</li> <li>• Selection of math process impaired</li> <li>• Difficulty determining salient information in word problems</li> </ul>

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## Executive Functioning and Mathematics

<u>EXECUTIVE DYSFUNCTION</u>	<u>BRAIN REGION</u>	<u>MATH SKILL</u>
<ul style="list-style-type: none"> <li>• <i>Organization Skills</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Dorsal-lateral PFC</i></li> </ul>	<ul style="list-style-type: none"> <li>• Inconsistent lining up math equations</li> <li>• Frequent erasers</li> <li>• Difficulty setting up problems</li> </ul>
<ul style="list-style-type: none"> <li>• <i>Self-Monitoring</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Dorsal-lateral PFC</i></li> </ul>	<ul style="list-style-type: none"> <li>• Limited double-checking of work</li> <li>• Unaware of plausibility to a response (<math>4 \times 6 = 137</math>)</li> </ul>

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## Three Basic Neural Codes to Format Numbers in the Brain

(1) Verbal Code - numbers are encoded as sequences of words (*twenty-four* instead of 24).  
- Dehaene & Cohen, 1997

- ▶ No need to understand quantitative concept.
- ▶ Main strategy used by younger children learning basic math facts (*two plus two equals four*)
- ▶ Critical for memorization of over-learned facts, such as multiplication facts (*nine times nine equals eighty-one*).
- ▶ Left perisylvian region of temporal lobes.

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### Three Basic Neural Codes to Format Numbers in the Brain

**(2) Procedural Code** - numbers are encoded as fixed symbols representing a quantity of some sort, and sequenced in a particular order. ( *24 instead of twenty-four*). - Von Aster, 2000

- I. Essential step to take numbers from a "word level" to a "quantitative level".
  - ▶ Circuitry involves the **syntactical arrangement of numerals** along our own internal number line. This requires an understanding of the **five** implicit rules of counting.
- II. Critical in the execution of mathematical **procedures** for equations not committed to rote memory (*i.e. subtraction with regrouping, long division, etc...*).
  - ▶ Bi-lateral occipital-temporal lobes (DeHaene, 1997).

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### 5 Rules of Counting (Geary, 2004)

- 1) **One-to-one correspondence** – one verbal tag given to each object.
- 2) **Stable order** – word tags are unchanged (invariant) across counted sets.
- 3) **Cardinality** – the value of the final word tag represents the total quantity.
- 4) **Abstraction** – any object can be counted.
- 5) **Order irrelevance** – can count in any order.

\* Rules of counting generally mastered by age 5.

\* Math LD kids in 2<sup>nd</sup> grade have a poor conceptual understanding of counting rules, and adhere to **adjacency rule** (belief that you must count objects in a linear order)

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### Three Basic Neural Codes to Format Numbers in the Brain

**(3) Magnitude Code** - numbers are encoded as analog quantities. Allows for value judgements, such as "9" is bigger than "4". (Chocon, et al, 1999)

- ▶ Allows for semantic understanding of math concepts and procedures....."**Number Sense**"
- ▶ Allows for the evaluation of the plausibility of a response. ( 9 X 4 = 94)
- ▶ Allows for the transcoding of more challenging tasks into palatable forms of operations. For instance, 15 percent of 80 becomes 10 percent of 80 plus half the value.
- ▶ Bi-lateral horizontal inferior parietal lobes.

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### 3 Subtypes of Math Disabilities

(1) Verbal Dyscalculia Subtype:

Main deficit is the automatic retrieval of number facts which have been stored in a linguistic code.

- ▶ Over-reliance on manipulatives when problem solving.
- ▶ Multiplication and addition often impaired.
- ▶ Poor at math fluency tests.
- ▶ Math algorithms often preserved.
- ▶ Often have learning disabilities in language arts as well.

**KEY CONSTRUCT:** Language & Verbal Retrieval

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### 3 Subtypes of Math Disabilities

(1) Verbal Dyscalculia Interventions:  
(Wright, Martland, & Stafford, 2000)



- ▶ Distinguish between reciting *number words*, and *counting* (words correspond to number concept).
- ▶ Develop a FNWS and BNWS to *ten*, *twenty*, and *thirty* without counting back. Helps develop an automatic retrieval skills (*Al's Game*, *Chris' Game*, *Chip's Game*)
- ▶ Develop a base-ten counting strategy whereby the child can perform addition and subtraction tasks involving tens and ones.
- ▶ Reinforce the language of math by re-teaching quantitative words such as *more*, *less*, *equal*, *sum*, *altogether*, *difference*, etc... (*April's Game*)

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### 3 Subtypes of Math Disabilities

(2) Procedural Dyscalculia Subtype:



- ▶ Difficulty recalling the algorithm or sequence of steps when performing longer math operations.
- ▶ Subtraction and division often impaired.
- ▶ Retrieval of math facts and magnitude comparisons often preserved.
- ▶ Only partial development of "number sense"

**Key Constructs:** Working Memory and Anxiety

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### 3 Subtypes of Math Disabilities

(2) **Procedural Dyscalculia Interventions:**

- ▶ Freedom from anxiety in class setting. Allow extra time for assignments and eliminate fluency drills.
- ▶ Color code math operational signs.
- ▶ Talk aloud all regrouping strategies.
- ▶ Use graph paper to line up equations.
- ▶ Adopt a curriculum such as “*Math Investigations*” which allows students to select their own algorithm.
- ▶ Attach number-line to desk and provide as many manipulatives as possible when problem solving.
- ▶ Teach skip-counting to learn multiplication facts.
- ▶ Teach patterns and relationships:  
( *Melissa's Game, Mama's Game, Cordelia's Game, Habib's Game* )

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### 3 Subtypes of Math Disabilities

(3) **Semantic Dyscalculia Subtype:**

A breakdown in comprehending magnitude representations between numbers and understanding the spatial properties of numeric relations. Can be associated with lower IQ and faulty executive functioning skills.

- ▶ Poor “number sense” and spatial attention.
- ▶ Difficulty evaluating the plausibility of a response (e.g.  $2 \times 4 = 24$ )
- ▶ Inability to transcode math operations into a more palatable form ( e.g.  $9 \times 4$  is same as  $(4 \times 10) - 4$  ).
- ▶ Poor magnitude comparisons.

**Key Constructs:** IQ, Executive Functioning, Visual-Spatial

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### 3 Subtypes of Math Disabilities

(3) **Semantic Dyscalculia Interventions:**

- ▶ Teach students to think in “*pictures*” as well as “*words*”.
- ▶ Have students explain their strategies when problem solving to expand problem solving options.
- ▶ Teach estimation skills to allow for effective previewing of response.
- ▶ Have students write a math sentence from a verbal sentence.
- ▶ Construct incorrect answers to equations and have students discriminate correct vs. incorrect responses.
- ▶ Incorporate money and measurement strategies to add relevance. Use “*baseball*” examples as well.
- ▶ ( *Heidi's Game, Dwain's Game* )

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### Intervention Summary

- (1) *Building number connections centered around a base-ten principle is crucial in the development of mathematical efficiency when problem solving.*
- (2) *Mathematical skill building and developing a conceptual understanding of quantitative knowledge should be fun, self-motivating, and require far less effort when presented in the format of games and activities.*
- (3) *In order to become facilitators of mathematical knowledge, students should practice multiple methods of problem solving by determining both a verbal and visual-spatial approach to solving addition, subtraction, multiplication, and division problems.*
- (4) *Math instruction should promote student directed algorithms and not teacher directed ones.*

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### National Mathematics Advisory Panel: Final Report (2008)

- ▶ Streamline the curriculum with precise math "focal points" bringing each topic to closure before moving on.
- ▶ Fractions are skill lacking the most in U.S. students.
- ▶ Conceptual understanding, procedural fluency, and problem solving skills are mutually reinforcing. Educators should focus on a balanced curricular approach.
- ▶ Automaticity of facts frees up working memory for more complex problem solving.
- ▶ Teachers math knowledge is important for student achievement.
- ▶ Explicit instruction for students should be provided for struggling math students.
- ▶ Mathematically gifted students should be allowed to accelerate their learning.
- ▶ Teach in a hierarchy from non-symbolic to symbolic to patterns and relationships to conceptual understanding.

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### The 90 Minute Mathematics' Assessment

- Intelligence Tests
- Visual-Spatial Functioning
- Working Memory Capacity
- Executive Functioning
- Attention Skills
- Math Skills and Number Sense
- Math Anxiety Scale
- Developmental and School History

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**Assessment Algorithm for Math:  
PAL II**

- Oral Counting
- Fact Retrieval (Look & Write- Listen & Say)
- Computational Operations
- Place Value
- Part-Whole Relationships
- Finding the Bug
- Multi-Step Problem Solving
- Numeral Writing
- Numeral Coding
- Quantitative & Spatial Working Memory
- Rapid Automatic Naming
- Fingertip Writing

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